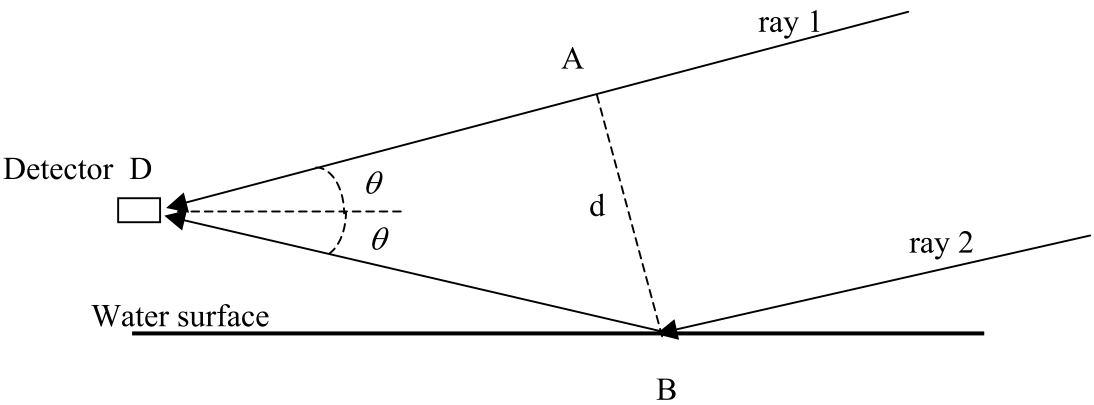
1. (optics) A microwave detector is located at the shore of a lake at a distance $d = 0.5$ m above the water level. The surface of the lake is totally reflective. A radio star emits monochromatic waves of wavelength $\lambda = 0.21$ m. When at the horizon, the phase difference between two parallel rays is π. Find the phase difference between two rays capture by the detector when the star is above the horizon of an angle $\alpha = 15$ degrees.

Hint: The phase difference to find is equal to the sum of two terms. The first term is given by the phase difference between two parallel rays. The second term needs to be calculated from reflective ray.

Given:
d = 0.5 m
λ = 0.21 m
θ = 15°

Solution:



Refer to the above ray diagram. According to the diagram, the path difference between ray 1 and ray 2 is given by the formula

$$\Delta r = \text{BD} - \text{AD} = \frac{d}{\sin 2\theta} - \frac{d}{\tan 2\theta} = \frac{d(1-\cos 2\theta)}{\sin 2\theta} = d \cdot \tan\theta$$

So the phase difference due to the path difference Δr is given by

$$\Delta\varphi_1 = \frac{2\pi}{\lambda}\Delta r = \frac{2\pi \cdot d}{\lambda}\tan\theta$$

The ray 2 is reflected from denser medium, so it changes the phase to an opposite one after reflection. Therefore the phase difference due reflection is given by

$$\Delta\varphi_2 = \pi$$

Thus the total phase difference between two rays is

$$\Delta\varphi = \Delta\varphi_1 + \Delta\varphi_2 = \frac{2\pi \cdot d}{\lambda}\tan\theta + \pi$$

Putting the numbers gives
$\Delta\varphi = (2*\pi*0.5/0.21)*\tan 15 + \pi = 7.15$ rad

2. (acoustics) A rope of total mass m and length L is suspended vertically. Find the time interval in which a transverse pulse travels the length, L, of the rope.

Given:
L, m
Find: Δt

Hint: For the rope problem, we have to show that: $\delta t = 2\sqrt{\frac{L}{g}}$

This problem is found using integration.

Solution:

The speed of transverse pulse in the rope, which is under tension with force F, is given by $v = \sqrt{\frac{F}{\mu}}$, where μ is a linear density of the rope, μ = m/L

Consider a small portion of the rope of length dy at distance y from the top. Then the tension in the portion is given by weight of the part of the rope under the portion

$F = \mu(L-y)g$

So the speed of transverse pulse in the portion is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{\mu(L-y)g}{\mu}} = \sqrt{(L-y)g}$$

The time the impulse travels across the portion is

$$dt = \frac{dt}{v} = \frac{dy}{\sqrt{(L-y)g}}$$

The total time interval in which the transverse pulse travels the total length of the rope is given by definite integral

$$\Delta t = \int_0^L dt = \int_0^L \frac{dy}{\sqrt{(L-y)g}}$$

Integrating gives

$$\Delta t = -\int_0^L \frac{d(L-y)}{\sqrt{(L-y)g}} = -\frac{1}{\sqrt{g}}\int_0^L \frac{d(L-y)}{\sqrt{L-y}} = -\frac{2}{\sqrt{g}}[\sqrt{L-y}]_0^L = 2\sqrt{\frac{L}{g}}.$$

3. (heat) An insulated Thermos contains 130 cm^3 of hot coffee, at a temperature of 80.0 degrees C. You put in a 12.0 g ice cube at its melting point to cool the coffee. By how many degrees has your coffee cooled once the ice has melted? Treat the coffee as though it were pure water and neglect energy transfers with the environment.

Given:
$V_1 = 130\ cm^3 = 1.30*10^{-4}\ m^3$
$T_1 = 80.0\ ^oC$
$m_2 = 12.0\ g = 1.20*10^{-2}\ kg$
$T_2 = 0\ ^oC$ is melting point of ice
$h = 3.34*10^5$ J/kg is specific heat melting of ice
$c = 4.186*10^3$ J/(kg*^{o}C) is specific heat of water
$d = 1.00*10^3\ kg/m^3$ is density of water

Solution:

Consider at first approximation that the coffee is water. Then the mass of the coffee is

$m_1 = V_1*d = 1.30*10^{-4}*1.00*10^3 = 0.130$ kg

Let T be final temperature of the coffee. Then we have the following heat balance equation

$c*m_1*(T_1 - T) = h*m_2 + c*m_2*(T - T_2)$

Solving for T gives

$T = (m_1*T_1 + m_2*T_2 - m_2*h/c)/(m_1+ m_2)$

Plugging in the numbers gives

$T = (0.130*80.0+0-1.20*10^{-2}*3.34*10^2/4.186)/(0.130+1.20*10^{-2}) = 66.50\ ^oC$

Thus once the ice has melted the coffee has been cooled by the following amount of degrees

$\Delta T = T_1 - T = 80.0-66.50 = 13.5\ ^oC$