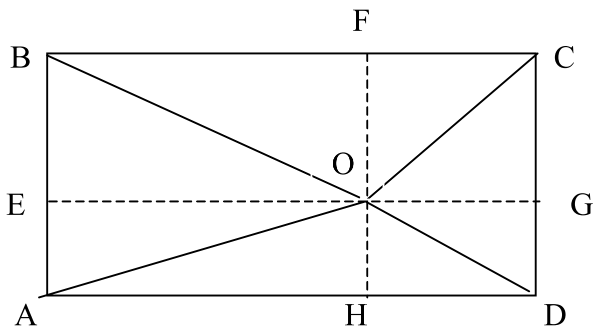
Given: ABCD is a rectangle and O is any point in the plane (within the rectangle).
Prove: $OA^2 + OC^2 = OB^2 + OD^2$

Solution:



Draw rectangle ABCD with arbitrary point O within it, and then draw lines OA, OB, OC, OD. Then draw lines from point O perpendicular to the sides: OE, OF, OG, OH.

Using Pythagorean theorem we have from the above diagram:
$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$
$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$
$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$
$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$

Adding these equalities we get:
$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$
$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$

From which we prove that for any point within the rectangle there is the relation
$OA^2 + OC^2 = OB^2 + OD^2$