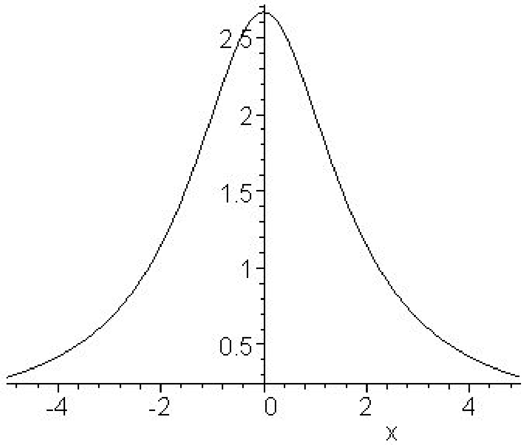
1) **Sketch the following curve: b(x)=8/(x^2+3). Use your knowledge of derivatives to determine any extrema, intercepts, points of inflection and asymptotes.**



Extrema: We take the drivative and equate it to zero:

$$b'(x) = -\frac{16x}{(x^2+3)^2} = 0$$

The point of extremum is x=0. We note that if x<0 then b'(x) >0 (function increases), if x>0 then b'(x) <0 (function decreases). This means the point x=0 is the point of maximum of the function b(x).

Intercepts: if x=0, b(0)= 8/3. This is the only intercept (with the y-axis). There are no intercepts with the x-axis, because the equation 8/(x^2+3)=0 has no solutions.

Points of inflection: We take the second derivative of b(x) and equate it to zero.

$$b''(x) = \frac{-16(x^2+3)^2 + 16x \cdot 4x(x^2+3)}{(x^2+3)^4} = 0,$$

This is equal to:

$$-16(x^2+3)^2 + 64x^2(x^2+3) = 0,$$

$$-16(x^2+3) + 64x^2 = 0,$$

$$-16x^2 - 48 + 64x^2 = 0,$$

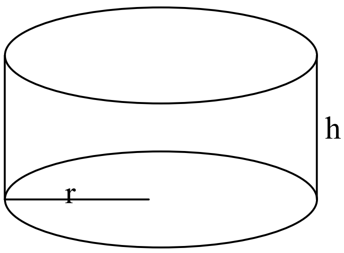
$$48x^2 = 48,$$

$$x = \pm 1 .$$

So, there are two points of inflection: x=1 and x= -1.

Asymptotes: There are no vertical asymptotes (because (x^2+3) can never be equal to zero), there is one horizontal asymptote y=0.

2) **A right circular cylinder is to be designed to hold 22 cu of a cola. The cost for the material for the top and bottom of the can is twice the cost for the material for the lateral surface. Find the dimensions of the cylinder that would minimize costs for the cola co.**



Let r be the radius of the cylinder, h is the height of the cylinder. Then the area of the top and bottom of the cylinder is $2\pi r^2$, area of the lateral surface is $2\pi rh$. Also, we know that the volume of the cylinder is 22, that is, $\pi r^2 \cdot h = 22$.

Let us denote the cost of the top and bottom surface unit as 2a, the cost of the lateral surface unit as a. Then the cost of the whole cylinder would be $2\pi r^2 \cdot 2a + 2\pi rh \cdot a$.

From the equation $\pi r^2 \cdot h = 22$ we find that $h = \frac{22}{\pi r^2}$.

Substituting it into the cost formula, we get

$$2\pi r^2 \cdot 2a + 2\pi r \frac{22}{\pi r^2} \cdot a \to \min$$

$$4\pi r^2 \cdot a + \frac{44}{r} \cdot a \to \min$$

Differentiating this function and equating it to zero:

$$8\pi r \cdot a - \frac{44}{r^2} \cdot a = 0,$$

Solving this equation with respect to r:

$$8\pi r = \frac{44}{r^2} \Rightarrow r^3 = \frac{44}{8\pi} \Rightarrow r^3 = \frac{11}{2\pi} \Rightarrow$$

$r = \sqrt[3]{\frac{11}{2\pi}}$ – **the sought radius.** (this approx. equals to 1.205)

$h = \frac{22}{\pi r^2} = \frac{22}{\pi\left(\frac{11}{2\pi}\right)^{2/3}}$ – **the sought height.** (this approx. equals to 4.821)

3) Use differentials to approximate $\sqrt[3]{124}$.

Here we should use the following formula:

$$\sqrt[3]{x + \Delta x} \approx \sqrt[3]{x} + \frac{\Delta x}{3\sqrt[3]{x^2}}.$$

This formula works for Δx, small enough in comparison with x.

We should apply this formula with the following values of x and Δx:

$x = 125, \quad \Delta x = -1$

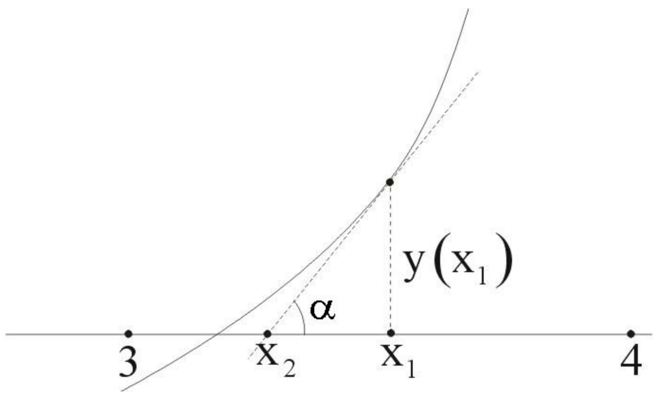
Substituting these values into the formula, we get:

$$\sqrt[3]{125 - 1} \approx \sqrt[3]{125} - \frac{1}{3\sqrt[3]{125^2}}$$

Simplifying it, we come to:

$$\sqrt[3]{124} \approx \sqrt[3]{125} - \frac{1}{3\sqrt[3]{125^2}} = 5 - \frac{1}{3 \cdot 25} \approx 4.9867.$$

4) **Use Newton method to find the root of** $y = x^2 - 2x - 5$ **on the interval** $3 < x < 4$**. Use** x_1**=3.5 to find** x_3**.**



The first step is to find x_2. Here is the description of the Newton Method:

We choose the first point in the interval where we do suspect to have the root. In our case they proposed to choose the middle point $x_1 = 3.5$. Also we note that the derivative of the function is equal to $y' = 2x - 2$.

Then we calculate the value of the function and its derivative in this point:

$$y(x_1) = x_1^2 - 2x_1 - 5 = 3.5^2 - 7 - 5 = 0.25$$

$$y'(x_1) = 2x_1 - 2 = 2 \cdot 3.5 - 2 = 5$$

Then we consider the following ratio:

$$\frac{y(x_1)}{x_1 - x_2} = \tan\alpha = y'(x_1)$$

$\tan\alpha$ is equal to $y'(x_1)$ according to the definition of the derivative (derivative is equal to the slope of the tangent line in the point $(x_1, y(x_1))$).

Then expressing x_2 from this formula we get:

$$x_2 = x_1 - \frac{y(x_1)}{y'(x_1)}$$

This is the formula for the second approximation of the root. Applying the same procedure now for the point x_2, we get the third approximation and so on… So the (n+1)-st approximation formula would be

$$x_{n+1} = x_n - \frac{y(x_n)}{y'(x_n)}$$

We need to find x_3. So we find x_2 first.

$$x_2 = x_1 - \frac{y(x_1)}{y'(x_1)} = 3.5 - \frac{0.25}{5} = 3.45$$

Then we calculate

$$y(x_2) = x_2^2 - 2x_2 - 5 = 3.45^2 - 6.9 - 5 = 0.0025,$$

$$y'(x_2) = 2x_2 - 2 = 2 \cdot 3.45 - 2 = 4.9$$

Then

$$x_3 = x_2 - \frac{y(x_2)}{y'(x_2)} = 3.45 - \frac{0.0025}{4.9} = 3.4494.$$

So, $x_3 = 3.4494$.